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$1/N$ expansion for exotic baryons

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Abstract: The $1/N_c$ expansion for exotic baryons is developed, and applied to the masses, meson couplings and decay widths. Masses and widths of the **27** and **35** pentaquark states in the same tower as the $\overline{\mathbf{10}} \supset \Theta^+$ are related by spin-flavor symmetry. The **27** and **35** states can decay within the pentaquark tower, as well as to normal baryons, and so have larger decay widths than the lightest pentaquark Θ. The $1/N_c$ expansion also is applied to baryon exotics containing a single heavy antiquark. The decay widths of heavy pentaquarks to normal heavy baryons via pion emission, and to normal baryons plus heavy $\overline{D}^{(*)}, B^{(*)}$ mesons are studied, and relations following from large-N_c spin-flavor symmetry and from heavy quark symmetry are derived.

Keywords: 1/N Expansion, QCD, Phenomenological Models.

Contents

1. Introduction

The recent discovery of the Θ^+ baryon with strangeness $S = +1$ [1] has led to renewed interest in hadron spectroscopy. The Θ^+ contains an $\bar{s}$ quark since it has strangeness $+1$, and is thought to be a five-quark $uudd\bar{s}$ state. The $q^4\bar{q}$ states which include the Θ^+ form a spin-1/2 flavor SU(3) antidecuplet. The states in the $\overline{\mathbf{10}}$ are shown in figure 1, using the tentative notation proposed by the Particle Data Group.

The states at the corners of the triangle are manifestly exotic, since they do not have the quantum numbers of qqq states. The observed $\Theta^+(1540)$ [1] is the isosinglet state at the top of the triangle. The $I = 3/2$ states $\Phi(1860)$ [2] (usually called $\Xi_{3/2}$) also have been observed and are thought to be the $ssqq\bar{q}$ members of the antidecuplet. The experimental situation is still confusing, with some experiments reporting discoveries, and others reporting upper limits on the production cross-section for the states. There is also an upper bound on the width of the Θ^+ derived in ref. [3]. In this article, we derive results for exotic baryons which can be obtained in a systematic expansion in $1/N_c$, where N_c is the number of colors. Readers not interested in technical details of the calculation can skip to the phenomenological results, which start at section 3.

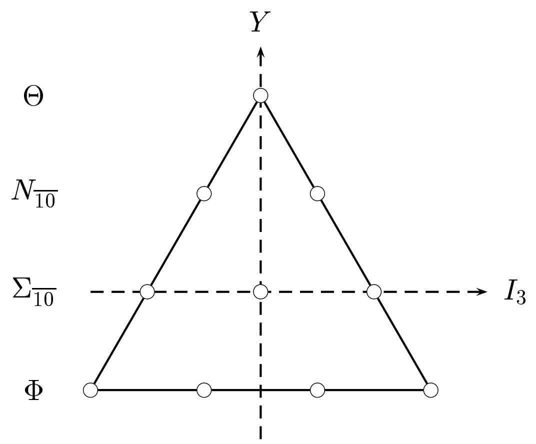

Figure 1: SU(3) weight diagram for the $\overline{\mathbf{10}}$ baryons.

The $1/N_c$ expansion of QCD [4] constrains the spin-flavor properties of baryons and their couplings to mesons. In the $N_c \to \infty$ limit, baryons have an exact contracted spin-flavor symmetry [5], which can be used to classify states. This spin-flavor symmetry is broken at subleading orders in $1/N_c$. The $1/N_c$ expansion is a systematic expansion for QCD, and gives results for the ground-state $[\mathbf{56}, 0^+]$ baryons ($\mathbf{8}_{1/2}$ and $\mathbf{10}_{3/2}$) in excellent agreement with experiment, with some predictions accurate to a fraction of a percent [5]–[10]. The $1/N_c$ expansion also has been successful in explaining the spin-flavor properties of heavy-quark baryons [11] and excited baryons [12]. In this paper, we extend the $1/N_c$ expansion to the lowest-lying exotic baryon states identified in ref. [13] using the method of quark operators [6, 7, 10]. We will concentrate on predictions in the isospin and SU(3) flavor symmetry limits for the exotic baryon masses, axial couplings and decay widths. A complete SU(3) analysis including SU(3)-breaking will be presented in a longer publication [14]. Implications of large-N_c for exotic baryons also have been considered in ref. [15].

In the $N_c \to \infty$ limit, baryons form irreducible representations of contracted spin-flavor symmetry $SU(6)_c$, and all states in a given irreducible representation (tower) are degenerate in the $N_c \to \infty$ limit [6]. For finite N_c, spin-flavor symmetry breaking generates a mass-splitting between the baryons in a given tower proportional to $\mathbf{J}^2/N_c$ [5]. The irreducible representations have been constructed as induced representations [6], and are referred to as the Skyrme representations, since their group-theoretic structure is identical to that in the Skyrme model. For finite N_c, it is convenient to work with irreducible representations of uncontracted spin-flavor symmetry SU(6), which are finite towers and are referred to as quark representations [6, 7, 10]. The quark and Skyrme representations are completely equivalent for $N_c \to \infty$ [16]. For finite N_c, the baryon tower of the Skyrme representation remains infinite and continues to satisfy the $SU(6)_c$ algebra. The quark representation gives a finite set of states for finite N_c, and satisfies the SU(6) algebra. QCD baryons satisfy the $SU(6)_c$ algebra for $N_c \to \infty$; the SU(6) algebra for finite N_c is not satisfied in QCD. The difference between Skyrme and quark representations is a subleading $1/N_c$ effect which can be absorbed into redefinitions of the unknown coefficients of $1/N_c$ suppressed operator products. For finite values of N_c, the two yield equivalent group theoretic results to any given order in $1/N_c$.

Large-N_c spin-flavor symmetry does not predict that exotic baryons exist. However, if exotic baryons do exist, consistency of the $1/N_c$ expansion relates the spin-flavor properties of exotic and non-exotic baryons at order $1/N_c$, just as it relates the spin-flavor properties of heavy-quark and light-quark baryons [11]. These same conclusions also are obtained in the Quark and Skyrme Models, which share the same contracted spin-flavor symmetry in the large-N_c limit. In the Quark Model, exotic baryons with exoticness[1] E form distinct irreducible representations of SU(6). In the Skyrme model, exotics contribute at $1/N_c$ [7], and cannot be separated from $1/N_c$ corrections. In both cases, one cannot prove that exotics must exist as $N_c \to \infty$.

[1]Exoticness is defined as the minimal value of E for which the baryon flavor representation can be constructed from $qqq(\bar{q}q)^E$. See ref. [13, 17] for a more extensive discussion, and for the definition of exoticness in the Skyrme model, which does not have explicit quark degrees of freedom.

In this paper, our main focus is on the lowest-lying baryon exotics, the pentaquarks. The $E = 1$ pentaquarks consist of the $\overline{\mathbf{10}}_{1/2}$, $\mathbf{27}_{1/2}$, $\mathbf{27}_{3/2}$, $\mathbf{35}_{3/2}$ and $\mathbf{35}_{5/2}$ representations of flavor and spin, and have positive parity. The results in this paper do not include SU(3) breaking. Thus, results for masses and widths are average values for all the states in a given SU(3) flavor representation. The $1/N_c$ expansion also can be performed using only isospin SU(2) flavor symmetry, with no assumption that the strange quark mass is small. The results obtained using isospin symmetry are similar to the results for three flavors in the limit of SU(3) symmetry. The SU(3) pentaquark towers decompose into isospin towers with fixed strangeness in the SU(2) flavor symmetry limit. The $S = 1$ states are the Θ^+ in the $\overline{\mathbf{10}}$, the isotriplets $\Theta_{1,j=1/2}$ and $\Theta_{1,j=3/2}$ in the $\mathbf{27}_{1/2}$ and $\mathbf{27}_{3/2}$, and the isotensors $\Theta_{2,j=3/2}$ and $\Theta_{2,j=5/2}$ in the $\mathbf{35}_{3/2}$ and $\mathbf{35}_{5/2}$. The individual isotriplet states are the Θ_1^{++}, Θ_1^{+} and Θ_1^{0}, and the individual isotensor states are Θ_2^{+++}, Θ_2^{++}, Θ_2^{+}, Θ_2^{0} and Θ_2^{-}. Unlike SU(3) flavor-symmetry breaking, isospin flavor-symmetry breaking can be neglected in comparison to $1/N_c$ corrections.

2. The $1/N_c$ expansion for exotics

Baryons of exoticness E have $N_c + E$ quarks in the completely symmetric $\mathrm{SU}(6)_q$ representation, and E antiquarks in the completely symmetric $\mathrm{SU}(6)_{\bar q}$ representation [13]. In a quark model description, E of the $N_c + E$ quarks must be in $\ell = 1$ orbitally excited wavefunctions, so that the total wavefunction satisfies Fermi statistics by being completely antisymmetric in color-orbital space [18]. Including the orbital angular momentum, the excited quarks q^* can be either $j = 1/2$ or $j = 3/2$. Whether the quark spin is actually made up of quark spin plus orbital angular momentum, or just plain quark spin, is irrelevant. All that matters for the spin-flavor algebra is the spin-flavor structure of the state. The irreducible representation we choose is the one with $q^*(j = 1/2)$ in a completely symmetric spin-flavor state with the other unexcited quarks. These states have positive parity. The other symmetry structures correspond to choosing different irreducible representations of $\mathrm{SU}(6)_c$. The mixed symmetry spin-flavor representation with all quarks in the ground state $\ell = 0$ wavefunction have negative parity [19]. Whether these are lighter is a dynamical issue. It has been argued that these are the lightest states for heavy pentaquarks [20].

The large-N_c spin-flavor algebra of baryon exotics can be written in terms of one-body quark and antiquark operators. Define bosonic quark and antiquark creation and annihilation operators $q^\dagger_{\mathcal{A}}$, $q^{\mathcal{A}}$, $\bar q^{\dagger\mathcal{A}}$ and $\bar q_{\mathcal{A}}$ with spin-flavor index $\mathcal{A} = 1, \ldots, 2F$ which satisfy the bosonic commutation relations $\left[q^{\mathcal{A}}, q^\dagger_{\mathcal{B}}\right] = \delta^{\mathcal{A}}_{\mathcal{B}}$ and $\left[\bar q_{\mathcal{A}}, \bar q^{\dagger\mathcal{B}}\right] = \delta^{\mathcal{B}}_{\mathcal{A}}$. The q and $\bar q$ creation and annihilation operators commute with one another. Note that the creation/annihilation operators of the quark representation do not carry color indices. Baryon exotics are color singlets, so it is not necessary to keep track of color for the spin-flavor analysis. The $\mathrm{SU}(2F)$ spin-flavor matrices in the $\Box$ and $\overline{\Box}$ representation are denoted by Λ^A and $\bar\Lambda^A = -(\Lambda^A)^T$, respectively, with normalization $\mathrm{Tr}\,\Lambda^A\Lambda^B = \delta^{AB}/2$. Under the decomposition $\mathrm{SU}(2F) \supset \mathrm{SU}(F) \otimes \mathrm{SU}(2)$ of the spin-flavor group to its flavor and spin subgroup, the $\mathrm{SU}(2F)$ fundamental index $\mathcal{A} \to \imath\alpha$, where $\imath =\uparrow\downarrow$ is a spin index and $\alpha = 1, \ldots, F$ is a flavor index. The SU(F) generators in the $\square$ and $\overline{\square}$ representations are $(T^a)^\alpha{}_\beta$ and $\left(\bar{T}^a\right)_\alpha{}^\beta$, respectively, with $\bar{T}^a = -(T^a)^T$ and $\mathrm{Tr}\, T^a T^b = \delta^{ab}/2$, and the spin generators are $\left(J^i\right)^{\imath}{}_{\jmath} = \left(\sigma^i\right)^{\imath}{}_{\jmath}/2$. [For SU(2), the $\bar{\mathbf{2}}$ is equivalent to the $\mathbf{2}$, so it is possible to also use J^i for the antiquark spin generators.]

The one-body quark and antiquark operators Λ_q and $\Lambda_{\bar{q}}$ are defined by

$$
\begin{aligned}
N_q &= q^\dagger q\,, & N_{\bar{q}} &= \bar{q}^\dagger \bar{q}\,,\\
J_q^i &= q^\dagger (J^i \otimes \mathbb{1}) q\,, & J_{\bar{q}}^i &= \bar{q}^\dagger \left(J^i \otimes \mathbb{1}\right) \bar{q}\,,\\
T_q^a &= q^\dagger (\mathbb{1} \otimes T^a) q\,, & T_{\bar{q}}^a &= \bar{q}^\dagger (\mathbb{1} \otimes \bar{T}^a) \bar{q}\,,\\
G_q^{ia} &= q^\dagger (J^i \otimes T^a) q\,, & G_{\bar{q}}^{ia} &= \bar{q}^\dagger (J^i \otimes \bar{T}^a) \bar{q}\,.
\end{aligned}
\tag{2.1}
$$

These one-body operators generate a $\mathrm{U}(6)_q \times \mathrm{U}(6)_{\bar{q}}$ spin-flavor algebra. $1/N_c$ effects break the separate quark and antiquark spin-flavor symmetries down to the diagonal SU(3) $\times$ SU(2) subgroup of SU(6). The properly normalized diagonal SU(6) generators are $(N_q - N_{\bar{q}})/(2\sqrt{F})$, $(J_q^i + J_{\bar{q}}^i)/\sqrt{F}$, $(T_q^a + T_{\bar{q}}^a)/\sqrt{2}$ and $\sqrt{2}(G_q^{ia} - G_{\bar{q}}^{ia})$. Baryons with exoticness E satisfy the identities $N_q = N_c + E$ and $N_{\bar{q}} = E$.

The hamiltonian (or other baryon observable) can be written as an expansion [6, 7, 10]

$$
H = \sum_n \frac{\mathcal{O}_n}{N_c^{n-1}}
\tag{2.2}
$$

where the summation is over all independent n-body operators $\mathcal{O}_n$. For baryon exotics, the independent n-body operators are monomials of order n in the basic one-body operators of eq. (2.1). Operator reduction rules [7] eliminate many operator products, considerably simplifying the $1/N_c$ expansion. For baryon exotics with quarks and antiquarks separately in completely symmetric spin-flavor representations, the quark operators satisfy the same identities given in ref. [7], with the replacement $N_c \to N_q = N_c + E$, and the antiquark operators separately satisfy the same identities with $N_q \to -N_{\bar{q}}$, $J_q^i \to J_{\bar{q}}^i$, $T_q^a \to T_{\bar{q}}^a$ and $G_q^{ia} \to -G_{\bar{q}}^{ia}$, and $N_c \to N_{\bar{q}} = E$.

Transition operators between baryons with different exoticness also are needed. The $\Delta E = -1$ one-body operators Λ_-^A which annihilate one $q\bar{q}$ pair are

$$
\begin{aligned}
N_- &= \bar{q} q\,, & T_-^a &= \bar{q} (\mathbb{1} \otimes T^a) q\,,\\
J_-^i &= \bar{q} (J^i \otimes \mathbb{1}) q\,, & G_-^{ia} &= \bar{q} (J^i \otimes T^a) q\,.
\end{aligned}
\tag{2.3}
$$

The hermitian conjugate $\Delta E = 1$ operators are $\Lambda_+^A = \Lambda_-^{A\dagger}$. The requirement that flavor-singlet quark-antiquark annihilation is forbidden for baryon exotics [13] implies that

$$
q^{\imath\alpha} \bar{q}_{\jmath\alpha} |E\rangle = 0\,, \qquad \langle E| q^\dagger_{\imath\alpha} \bar{q}^{\dagger \jmath\alpha} = 0
\tag{2.4}
$$

where $|E\rangle$ denotes a baryon state with exoticness E. States which satisfy eq. (2.4) will be called minimal states, and we will restrict our analysis to these states. Non-minimal states can be included by treating them as minimal states plus a $\bar{q}q$ pair [21]. Eq. (2.4) implies that for minimal states

$$
N_- = 0\,, \qquad J_-^i = 0\,, \qquad N_+ = 0\,, \qquad J_+^i = 0\,.
\tag{2.5}
$$

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Operator products of Λ_- and Λ_+ do not need to be considered, since these operators can be rewritten as linear combinations of Λ_q and $\Lambda_{\bar{q}}$. Operators in a given ΔE sector are products of the form $(\Lambda_\pm)^{|\Delta E|}$ times monomials in Λ_q and $\Lambda_{\bar{q}}$.

There are new operator identities which reduce the number of independent operators products of the form $\Lambda_q\Lambda_{\bar{q}}$, $\Lambda_q\Lambda_-$, $\Lambda_{\bar{q}}\Lambda_-$ and $\Lambda_-\Lambda_-$, as well as the hermitian conjugate products involving Λ_+ [14]. The identities for an SU(2) analysis correspond to a simple operator reduction rule: *Operator products in which two flavor indices are contracted using δ^{ab} or ϵ^{abc} can be eliminated.*

In the SU(3) limit, the most general $\Delta E = 0$ hamiltonian is a polynomial in the one-body operators divided by N_c. The operator reduction rules imply that H has the form

$$H_0 = N_c f\left(\frac{J_q^2}{N_c^2}, \frac{J_{\bar{q}}^2}{N_c^2}, \frac{J^2}{N_c^2}, \frac{E}{N_c}\right), \tag{2.6}$$

where the function f is a general polynomial of its arguments. The hamiltonian to order $1/N_c$ is

$$H_0 = c_0 N_c + c_1 E + \frac{1}{N_c}\left(c_2 J_q^2 + c_3 J_{\bar{q}}^2 + c_4 J^2 + c_5 E^2\right) \tag{2.7}$$

where the coefficients c_i are functions of $1/N_c$ of order N_c^0, and exoticness E is assumed to be order one. Although the hamiltonian is symmetric under $q \leftrightarrow \bar{q}$, this symmetry is no longer manifest in eq. (2.7) because the substituted values N_q and $N_{\bar{q}}$ are asymmetric. For example,

$$\frac{1}{N_c^2}\left[N_q J_q^2 + N_{\bar{q}} J_{\bar{q}}^2\right] \tag{2.8}$$

which is symmetric under $q \leftrightarrow \bar{q}$ becomes

$$\frac{1}{N_c^2}\left[(N_c + E) J_q^2 + E J_{\bar{q}}^2\right] \to \frac{1}{N_c} J_q^2 \tag{2.9}$$

to order $1/N_c$ (assuming E is order one). As a result, c_2 and c_3 in eq. (2.7) need not be equal.

The hamiltonian in the Skyrme Model is given by eq. (2.7) with the additional constraints [13]

$$c_2 = c_3 = 2c_5 = 2c_1\,, \tag{2.10}$$

if collective coordinate quantization with neglect of vibrational-rotational coupling is used. Eq. (2.10) does not follow from a general $1/N_c$ analysis. The computation of the Θ^+ mass in ref. [22] used this restricted form for the hamiltonian, but it has been shown that the computation in ref. [22] is not a consistent semiclassical computation [23, 24, 25]. Including vibrational-rotational couplings gives a hamiltonian which does not satisfy eq. (2.10). The results in this paper use the general form of the $1/N_c$ hamiltonian, and do not rely on eq. (2.10).

In the SU(3) flavor symmetry limit, the most general $\Delta E \neq 0$ hamiltonian vanishes, and there is no mixing between states with different ΔE. To first order in SU(3) breaking,

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the hamiltonian is given by constructing all possible independent spin-singlet terms which transform as T^8. The $\Delta E = 0$ hamiltonian is

$$
\begin{aligned}
H^8 &= c_1^{(8)} T_q^8 + c_2^{(8)} T_{\bar{q}}^8 + \frac{E}{N_c}(c_3^{(8)} T_q^8 + c_4^{(8)} T_{\bar{q}}^8) + \\
&+ \frac{1}{N_c}\left[c_5^{(8)} J_q^i G_q^{i8} + c_6^{(8)} J_q^i G_{\bar{q}}^{i8} + c_7^{(8)} J_{\bar{q}}^i G_q^{i8} + c_8^{(8)} J_{\bar{q}}^i G_{\bar{q}}^{i8}\right] \qquad (2.11)
\end{aligned}
$$

to order $1/N_c$. This form of the $1/N_c$ expansion requires operator identities for the SU(3) analysis, which are deferred to a future publication. Similarly, the $\Delta E = -1$ hamiltonian is

$$
H_-^8 = d_1^{(8)} T_-^8 + \frac{1}{N_c}\left[d_2^{(8)} E T_-^8 + d_3^{(8)} J_q^i G_-^{i8}\right]. \qquad (2.12)
$$

The $\Delta E = -1$ hamiltonian mixes the pentaquark states with the ordinary $E = 0$ baryon states at order $m_s/\sqrt{N_c}$. The $1/\sqrt{N_c}$ suppression factor is obtained because the matrix elements of Λ_- operators are at most of order $\sqrt{N_c}$.

The $\Delta E = 0$ axial currents are given by

$$
\frac{1}{2}A^{ia} = g_A G_q^{ia} + g_1 G_{\bar{q}}^{ia} + g_2 \frac{\{E, G_q^{ia}\}}{N_c} + g_3 \frac{\{E, G_{\bar{q}}^{ia}\}}{N_c} + g_4 \frac{J_q^i T_q^a}{N_c} + g_5 \frac{J_{\bar{q}}^i T_q^a}{N_c} + g_6 \frac{J_q^i T_{\bar{q}}^a}{N_c} + g_7 \frac{J_{\bar{q}}^i T_{\bar{q}}^a}{N_c} \qquad (2.13)
$$

up to operator products of relative order $1/N_c^2$. The leading term G_q^{ia} is order N_c for $a = 1, 2, 3$, order $\sqrt{N_c}$ for $a = 4, 5, 6, 7$, and order 1 for $a = 8$. Relative to this leading term, the terms g_1, g_2, g_4 and g_5 are order $1/N_c$ and the terms g_3, g_6 and g_7 are order $1/N_c^2$. The coupling g_A is normalized to the axial coupling of a constituent quark, so that $g_A(n \to p) = 1.25 = 5g_A/3$ gives $g_A \sim 0.75$ [26]. To leading order the axial pion octet couplings in the $E = 0$ nucleon tower and $E = 1$ exotic tower are equal; they differ at relative order $1/N_c$ due to g_1, g_2, g_4 and g_5.

The $\Delta E = -1$ axial currents are given by

$$
\frac{1}{2}A_-^{ia} = \bar{g}_0 G_-^{ia} + \bar{g}_1 \frac{\{E, G_-^{ia}\}}{N_c} + \bar{g}_3 \frac{J_q^i T_-^a}{N_c} + \bar{g}_4 \frac{J_{\bar{q}}^i T_-^a}{N_c} \qquad (2.14)
$$

to relative order $1/N_c$. The leading term G_-^{ia} is order $\sqrt{N_c}$ for $a = 1, 2, 3$, order 1 for $a = 4, 5, 6, 7$, and order $1/\sqrt{N_c}$ for $a = 8$, so these transition couplings are suppressed by $1/\sqrt{N_c}$ relative to the $\Delta E = 0$ axial couplings, such as $g_{NN\pi}$, discussed above. The $1/N_c$ expansion of the $\Delta E = -1$ axial couplings relates the pion octet couplings between pentaquark and non-exotic baryons. All of the decay couplings between the $E = 1$ and $E = 0$ baryons are given by a single unknown coupling $\bar{g}_0$ up to corrections of relative order $1/N_c$. A quantitative measure of the suppression of $\Delta E = -1$ couplings is given by the ratio $\bar{g}_0/g_A$.

3. Mass relations

A detailed discussion of the above equations including SU(3) breaking will be given in longer publications [14, 27]. Here we limit ourselves to a discussion of the mass relations and widths in the SU(3) flavor symmetry limit. The hamiltonian eq. (2.7) yields, by taking linear combinations to eliminate the unknown coefficients c_{0-5}, the mass relations

$$
\begin{aligned}
2\left(\overline{\mathbf{10}}_{1/2}\right)+\langle\mathbf{35}\rangle &= 3\langle\mathbf{27}\rangle+\mathcal{O}\left(\frac{1}{N_c^3}\right),\\
\left(\mathbf{35}_{5/2}-\mathbf{35}_{3/2}\right) &= \frac{5}{3}\left(\mathbf{27}_{3/2}-\mathbf{27}_{1/2}\right)+\mathcal{O}\left(\frac{1}{N_c^3}\right),\\
\langle\mathbf{27}\rangle-\overline{\mathbf{10}}_{1/2} &= \frac{2}{3}\left(\mathbf{10}_{3/2}-\mathbf{8}_{1/2}\right)+\mathcal{O}\left(\frac{1}{N_c^2}\right),\\
\langle\mathbf{35}\rangle-\overline{\mathbf{10}}_{1/2} &= 2\left(\mathbf{10}_{3/2}-\mathbf{8}_{1/2}\right)+\mathcal{O}\left(\frac{1}{N_c^2}\right),
\end{aligned}
\tag{3.1}
$$

where

$$\langle\mathbf{27}\rangle \equiv \frac{1}{3}\left[\left(\mathbf{27}_{1/2}\right)+2\left(\mathbf{27}_{3/2}\right)\right], \tag{3.2}$$

$$\langle\mathbf{35}\rangle \equiv \frac{1}{5}\left[2\left(\mathbf{35}_{3/2}\right)+3\left(\mathbf{35}_{5/2}\right)\right] \tag{3.3}$$

denote spin-averaged masses. The fourth relation is not linearly independent. The first two relations also are satisfied by the $1/N_c^2$ terms in the hamiltonian, and so are true to $1/N_c^3$.

Using only isospin SU(2) flavor symmetry, one gets the corresponding relations for the $S=1$ states:

$$
\begin{aligned}
2\Theta^+ +\langle\Theta_2\rangle &= 3\langle\Theta_1\rangle+\mathcal{O}\left(\frac{1}{N_c^3}\right),\\
\left(\Theta_{2,j=5/2}-\Theta_{2,j=3/2}\right) &= \frac{5}{3}\left(\Theta_{1,j=3/2}-\Theta_{1,j=1/2}\right)+\mathcal{O}\left(\frac{1}{N_c^3}\right),\\
\langle\Theta_1\rangle-\Theta^+ &= \frac{2}{3}(\Delta-N)+\mathcal{O}\left(\frac{1}{N_c^2}\right)=195\,\mathrm{MeV}+\mathcal{O}\left(\frac{1}{N_c^2}\right),\\
\langle\Theta_2\rangle-\Theta^+ &= 2(\Delta-N)+\mathcal{O}\left(\frac{1}{N_c^2}\right)=58\,\mathrm{MeV}+\mathcal{O}\left(\frac{1}{N_c^2}\right),
\end{aligned}
\tag{3.4}
$$

where

$$\langle\Theta_1\rangle \equiv \frac{1}{3}\left(\Theta_{1,j=1/2}+2\Theta_{1,j=3/2}\right), \tag{3.5}$$

$$\langle\Theta_2\rangle \equiv \frac{1}{5}\left(2\Theta_{2,j=3/2}+3\Theta_{2,j=5/2}\right), \tag{3.6}$$

denote spin-averaged masses. These relations do not rely on the assumption of approximate SU(3) flavor symmetry, and they are identical to the mass relations obtained in ref. [23] in the Skyrme model. The $1/N_c$ analysis given here is model-independent and gives the accuracy of each mass relations as a power of $1/N_c$. The $1/N_c^3$ and $1/N_c^2$ mass relations are expected to hold at the 10 MeV and 30 MeV level since the leading order N_c mass is approximately 1 GeV. Using $\Theta^+ = 1540$ MeV, these relations imply $\langle\Theta_1\rangle = 1735$ MeV and $\langle\Theta_2\rangle = 2126$ MeV, to an accuracy of $1/N_c^2$, or approximately 30 MeV. One cannot determine the Θ^+ mass because of the order $N_c^0 = 1$ term linear in E in the hamiltonian which splits the exotic $E = 1$ baryon masses from the normal $E = 0$ baryon masses.[2]

4. Decay widths

The decay widths can be computed to order $1/N_c$ using eqs. (2.13), (2.14) for the pion octet couplings. The exotic and ground state baryons have the same parity, so the decays are in the odd partial waves, since the π and K are pseudoscalars. The $j = 1/2 \leftrightarrow j = 1/2$ and $j = 1/2 \leftrightarrow j = 3/2$ transitions are only p-wave, by angular momentum conservation. All the decays are dominantly p-wave decays as long as the momentum of the final particles is small. Transitions between normal baryons or between $E = 1$ baryons are $\Delta E = 0$ transitions, and depend on g_A. Transitions between the $E = 1$ tower and the $E = 0$ tower are $\Delta E = -1$ transitions, and depend on $\bar{g}_0$ (see figure 2).

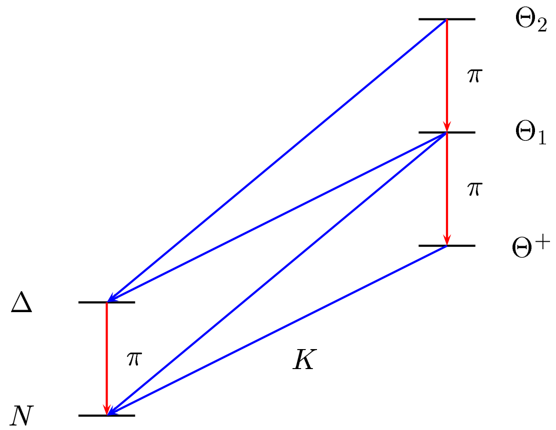

Figure 2: Transitions between the pentaquark and normal baryon towers. The transitions within a tower (vertical red lines) depend on g_A, and those between towers (diagonal blue lines) depend on $\bar{g}_0$. The Θ^+ is naturally much narrower than the excited pentaquarks, since it is the lightest pentaquark and cannot decay within the pentaquark tower.

The $\Delta E = 0$ transitions can be normalized to $\Delta \to N\pi$, and the $\Delta E = -1$ transitions can be normalized to $\Theta \to NK$. The decay widths all have a p^3 phase space dependence, where p is the momentum of the emitted pseudoscalar meson. The ratios of decay widths correcting for the p^3 phase space dependence (i.e. the ratios of the transition amplitudes squared) are:

$$\frac{p^{-3}\Gamma(\Theta_{1,j=1/2} \to NK)}{p^{-3}\Gamma(\Theta \to NK)} = \frac{2}{9},$$
$$\frac{p^{-3}\Gamma(\Theta_{1,j=3/2} \to NK)}{p^{-3}\Gamma(\Theta \to NK)} = \frac{8}{9},$$
$$\frac{p^{-3}\Gamma(\Theta_{1,j=1/2} \to \Delta K)}{p^{-3}\Gamma(\Theta \to NK)} = \frac{4}{9},$$

[2] The decay widths $\Theta \to NK$ are of order N_c^0 [28, 13] but do not destroy the above mass relations. Decay couplings are constrained by the $1/N_c$ expansions eq. (2.13), (2.14), and yield decay widths which satisfy the same $1/N_c$ relations as the masses. Mixing between exotic and ordinary baryons also does not destroy the hierarchy of baryon mass predictions for the ordinary baryons, including the successful Gell-Mann–Okubo relations. The mixing hamiltonian eq. (2.14) is constrained by the $1/N_c$ expansion, and produces mass shifts which satisfy the $1/N_c$ relations in ref. [9]. In addition, it is known for ground-state baryons that radiative corrections such as chiral loops respect the $1/N_c$ relations [6, 29]. This result should not be a surprise—the $1/N_c$ relations are *symmetry* relations which are preserved by all interactions in the theory.

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$$\begin{aligned}
\frac{p^{-3}\Gamma(\Theta_{1,j=3/2} \to \Delta K)}{p^{-3}\Gamma(\Theta \to NK)} &= \frac{5}{18}\,, \\
\frac{p^{-3}\Gamma(\Theta_{2,j=3/2} \to \Delta K)}{p^{-3}\Gamma(\Theta \to NK)} &= \frac{1}{2}\,, \\
\frac{p^{-3}\Gamma(\Theta_{2,j=5/2} \to \Delta K)}{p^{-3}\Gamma(\Theta \to NK)} &= \frac{4}{3}\,, \\
\frac{p^{-3}\Gamma(\Theta_{1,j=1/2} \to \Theta\pi)}{p^{-3}\Gamma(\Delta \to N\pi)} &= \frac{4}{3}\,, \\
\frac{p^{-3}\Gamma(\Theta_{1,j=3/2} \to \Theta\pi)}{p^{-3}\Gamma(\Delta \to N\pi)} &= \frac{4}{3}\,, \\
\frac{p^{-3}\Gamma(\Theta_{2,j=3/2} \to \Theta_{1,j=1/2}\pi)}{p^{-3}\Gamma(\Delta \to N\pi)} &= \frac{5}{4}\,, \\
\frac{p^{-3}\Gamma(\Theta_{2,j=3/2} \to \Theta_{1,j=3/2}\pi)}{p^{-3}\Gamma(\Delta \to N\pi)} &= \frac{1}{4}\,, \\
\frac{p^{-3}\Gamma(\Theta_{2,j=5/2} \to \Theta_{1,j=3/2}\pi)}{p^{-3}\Gamma(\Delta \to N\pi)} &= \frac{3}{2}\,.
\end{aligned} \tag{4.1}$$

The pion decay ratios have corrections of order $1/N_c$, since they involve transitions in two different towers. The corrections are of order $1/N_c^2$ if one takes ratios of pion decays between different states in the pentaquark tower, e.g. $p^{-3}\Gamma(\Theta_{1,j=1/2} \to \Theta\pi)/p^{-3}\Gamma(\Theta_{1,j=3/2} \to \Theta\pi) = 1 + \mathcal{O}(1/N_c^2)$, by taking the ratio of two pion entries in eq. (4.1). The kaon decay ratios have corrections of order $1/N_c$.

The Δ width is [30]

$$\Gamma(\Delta \to N\pi) = \frac{g_A^2 p^3}{3\pi f_\pi^2}\,, \tag{4.2}$$

which normalizes all of the $\Delta E = 0$ decays. The Θ width is

$$\Gamma(\Theta^+) = \Gamma(\Theta^+ \to NK) = \frac{\bar{g}_0^2 p^3}{2\pi f_K^2} = 10\,\mathrm{MeV}\left(\frac{\bar{g}_0}{0.2}\right)^2, \tag{4.3}$$

which shows that $\bar{g}_0/g_A$ must be at least a factor of four smaller than g_A for a Θ^+ width below 10 MeV. The coupling between different towers $\bar{g}_0$ is expected to be smaller than the coupling within a given tower g_A, $\bar{g}_0 < g_A$. An example of such a suppression is the comparison of the coupling between the excited nucleon and ground state nucleon towers: $g(N^*N\pi)^2/g(NN\pi)^2 \sim 1/10$ [30].

The $j = 3/2$ and $5/2$ isotensors Θ_2 and the $j = 1/2$ and $3/2$ isotriplets Θ_1 can decay via $\Delta E = 0$ pion emission to lower states in the $E = 1$ tower; the Θ, being the lightest $E = 1$ state with $S = 1$, can only decay via a $\Delta E = -1$ transition, as show in figure 2. Transitions within a tower are controlled by g_A and between towers by $\bar{g}_0$, so that naturally $\Gamma(\Theta) \ll \Gamma(\Theta_1), \Gamma(\Theta_2)$. Using 1735 MeV for both the Θ_1 states, and 2126 MeV for both the Θ_2 states gives $\Gamma(\Theta_{1,j=1/2}) \gtrsim 30\,\mathrm{MeV} + 1.2\Gamma(\Theta)$, $\Gamma(\Theta_{1,j=3/2}) \gtrsim 30\,\mathrm{MeV} +$

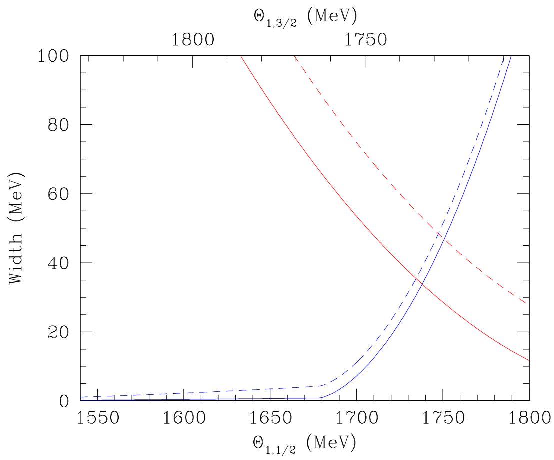

Figure 3: Width of the Θ_1 states as a function of the Θ_1 mass. The blue curves are for $j = 1/2$, and the red curves for $j = 3/2$. The solid and dashed curves are for a Θ width of 1 and 5 MeV, respectively.

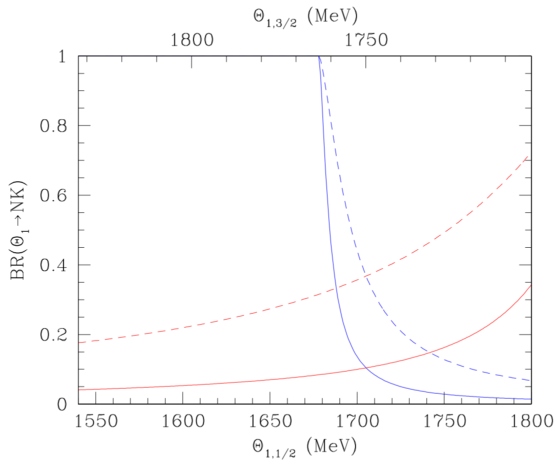

Figure 4: Branching ratio of the Θ_1 states into NK as a function of the Θ_1 mass. The blue curves are for $j = 1/2$, and the red curves for $j = 3/2$. The solid and dashed curves are for a Θ width of 1 and 5 MeV, respectively.

$4.9\Gamma(\Theta)$, $\Gamma(\Theta_{2,j=3/2}) \gtrsim 560\,\text{MeV} + 5.1\Gamma(\Theta)$ and $\Gamma(\Theta_{2,j=5/2}) \gtrsim 560\,\text{MeV} + 13.6\Gamma(\Theta)$.[3] The isotensor **35** states are seen to be extremely broad, and will not be distinguishable from the continuum.

The widths of the isotriplet flavor-**27** states depend on the value of the hyperfine splitting between the $j = 1/2$ and $j = 3/2$ states. Since the hyperfine splitting does not change the average mass, one isotriplet state will have higher mass (and width), and the other lower mass (and width) than the results quoted using the average mass. In figures 3, 4 we have plotted the decay widths and NK branching ratio of the states $\Theta_{1,j=1/2}$ and $\Theta_{1,j=3/2}$ as a function of their mass. The mass of the $j = 1/2$ state is given by the lower axis, and the mass of the $j = 3/2$ state by the upper axis. Note that the two masses are correlated — the spin-averaged mass of the two states has been set to 1735 MeV. The variation with mass is entirely due to the p^3 phase space dependence of the widths. The rapid rise in the $j = 1/2$ width (and fall in its NK branching ratio) around 1700 MeV is due to onset of the $\Theta\pi$ decay channel. The Θ_1 states are wide as long as they are above $\Theta\pi$ threshold.

5. Heavy pentaquarks

One can have $qqqq\bar{Q}$ exotics containing heavy quarks, which are analogs of the Θ and related states. We will study the masses and decays of these heavy pentaquarks in this section. Note that $qqqQ\bar{q}$ exotics [31], as well as exotics containing more than one heavy quark, also could exist, but are not discussed here.

[3] A note of caution: The $\Theta_{1,j=3/2} - \Theta_{1,j=1/2}$ mass splitting is not included in the numerical values quoted. The p^3 phase-space factor depends strongly on the masses. The results given are inequalities because there may be other channels (such as K decays between pentaquarks) which can exist if the mass splittings allow.

5.1 Masses

One can derive mass relations for the lowest-lying $qqqq\bar{Q}$ baryons ($Q = c, b$), which consist of the $\overline{\mathbf{6}}_{1/2}$, $\mathbf{15}_{1/2}$, $\mathbf{15}_{3/2}$, $\mathbf{15'}_{3/2}$ and $\mathbf{15'}_{5/2}$ multiplets. The mass relations in the SU(3) flavor symmetry limit are

$$\begin{aligned}
2\left(\overline{\mathbf{6}}_{1/2}\right) + \langle \mathbf{15'} \rangle &= 3\langle \mathbf{15} \rangle + \mathcal{O}\left(\frac{1}{N_c^3}\right), \\
\left(\mathbf{15'}_{5/2} - \mathbf{15'}_{3/2}\right) &= \frac{5}{3}\left(\mathbf{15}_{3/2} - \mathbf{15}_{1/2}\right) + \mathcal{O}\left(\frac{1}{(m_Q N_c^3)}\right), \\
\langle \mathbf{15} \rangle - \overline{\mathbf{6}}_{1/2} &= \frac{2}{3}(\mathbf{10}_{3/2} - \mathbf{8}_{1/2}) + \mathcal{O}\left(\frac{1}{N_c^2}\right), \\
\langle \mathbf{15'} \rangle - \overline{\mathbf{6}}_{1/2} &= 2(\mathbf{10}_{3/2} - \mathbf{8}_{1/2}) + \mathcal{O}\left(\frac{1}{N_c^2}\right),
\end{aligned} \tag{5.1}$$

where

$$\langle \mathbf{15} \rangle = \frac{1}{3}\left[\left(\mathbf{15}_{1/2}\right) + 2\left(\mathbf{15}_{3/2}\right)\right], \tag{5.2}$$

$$\langle \mathbf{15'} \rangle = \frac{1}{5}\left[2\left(\mathbf{15'}_{3/2}\right) + 3\left(\mathbf{15'}_{5/2}\right)\right], \tag{5.3}$$

denote spin-averaged masses and the $\mathbf{6}_{1/2}$ and $\mathbf{6}_{3/2}$ are the non-exotic qqQ baryon multiplets containing the $\Sigma_{c,b}$ and $\Sigma^*_{c,b}$. The $\left(\mathbf{15}_{3/2} - \mathbf{15}_{1/2}\right)$ and $\left(\mathbf{15'}_{5/2} - \mathbf{15'}_{3/2}\right)$ mass splittings violate heavy quark spin-symmetry and are proportional to $1/m_Q$. The above mass relations are identical to those for the light pentaquarks, since the hamiltonian has the same structure as eq. (2.7). The only difference is that the hyperfine splittings, which violate heavy quark spin symmetry, are suppressed by $1/m_Q$.

For two light flavors, the $qqqq\bar{Q}$ baryon exotics consist of the isosinglet $\Theta_{\bar{Q}}$ in the $\overline{\mathbf{6}}_{1/2}$, the isotriplets $\Theta_{1,\bar{Q},j=1/2}$ and $\Theta_{1,\bar{Q},j=3/2}$ in the $\mathbf{15}_{1/2}$ and $\mathbf{15}_{3/2}$, and the isotensors $\Theta_{2,\bar{Q},j=3/2}$ and $\Theta_{2,\bar{Q},j=5/2}$ in the $\mathbf{15'}_{3/2}$ and $\mathbf{15'}_{5/2}$. Using only isospin SU(2) flavor symmetry, one obtains the mass relations

$$\begin{aligned}
2\left(\Theta_{\bar{Q}}\right) + \langle \Theta_{2,\bar{Q}} \rangle &= 3\langle \Theta_{1,\bar{Q}} \rangle + \mathcal{O}\left(\frac{1}{N_c^3}\right), \\
\left(\Theta_{2,\bar{Q},j=5/2} - \Theta_{2,\bar{Q},j=3/2}\right) &= \frac{5}{3}\left(\Theta_{1,\bar{Q},j=3/2} - \Theta_{1,\bar{Q},j=1/2}\right) + \mathcal{O}\left(\frac{1}{(m_Q N_c^3)}\right), \\
\langle \Theta_{1,\bar{Q}} \rangle - \Theta_{\bar{Q}} &= \frac{2}{3}(\Delta - N) + \mathcal{O}\left(\frac{1}{N_c^2}\right), \\
\langle \Theta_{2,\bar{Q}} \rangle - \Theta_{\bar{Q}} &= 2(\Delta - N) + \mathcal{O}\left(\frac{1}{N_c^2}\right),
\end{aligned} \tag{5.4}$$

where

$$\langle \Theta_{1,\bar{Q}} \rangle \equiv \frac{1}{3}\left(\Theta_{1,\bar{Q},j=1/2} + 2\Theta_{1,\bar{Q},j=3/2}\right), \tag{5.5}$$

$$\langle \Theta_{2,\bar{Q}} \rangle \equiv \frac{1}{5}\left(2\Theta_{2,\bar{Q},j=3/2} + 3\Theta_{2,\bar{Q},j=5/2}\right), \tag{5.6}$$

denote spin-averaged masses. These mass relations imply

$$
\begin{aligned}
\Theta_{1,\bar{Q},j=1/2} &= \Theta_{\bar{Q}} + \frac{2}{3}(\Delta - N) - \frac{2}{3}\delta\Theta_{1,\bar{Q}} + \mathcal{O}\left(\frac{1}{N_c^2}\right), \\
\Theta_{1,\bar{Q},j=3/2} &= \Theta_{\bar{Q}} + \frac{2}{3}(\Delta - N) + \frac{1}{3}\delta\Theta_{1,\bar{Q}} + \mathcal{O}\left(\frac{1}{N_c^2}\right), \\
\Theta_{2,\bar{Q},j=3/2} &= \Theta_{\bar{Q}} + 2(\Delta - N) - \delta\Theta_{1,\bar{Q}} + \mathcal{O}\left(\frac{1}{N_c^2}\right), \\
\Theta_{2,\bar{Q},j=5/2} &= \Theta_{\bar{Q}} + 2(\Delta - N) + \frac{2}{3}\delta\Theta_{1,\bar{Q}} + \mathcal{O}\left(\frac{1}{N_c^2}\right),
\end{aligned}
\tag{5.7}
$$

where $\delta\Theta_{1,\bar{Q}} = \left(\Theta_{1,\bar{Q},j=3/2} - \Theta_{1,\bar{Q},j=1/2}\right)$ is the heavy quark spin-symmetry violating hyperfine splitting proportional to $(J_{\bar{Q}} \cdot J_q)/N_c m_Q$ [11]. Substituting in the numerical value for the $(\Delta - N)$ mass difference gives

$$
\begin{aligned}
\Theta_{1,\bar{c},j=1/2} &= \Theta_{\bar{c}} + 195\,\mathrm{MeV} - \frac{2}{3}\delta\Theta_{1,\bar{c}} + \mathcal{O}\left(\frac{1}{N_c^2}\right), \\
\Theta_{1,\bar{c},j=3/2} &= \Theta_{\bar{c}} + 195\,\mathrm{MeV} + \frac{1}{3}\delta\Theta_{1,\bar{c}} + \mathcal{O}\left(\frac{1}{N_c^2}\right), \\
\Theta_{2,\bar{c},j=3/2} &= \Theta_{\bar{c}} + 586\,\mathrm{MeV} - \delta\Theta_{1,\bar{c}} + \mathcal{O}\left(\frac{1}{N_c^2}\right), \\
\Theta_{2,\bar{c},j=5/2} &= \Theta_{\bar{c}} + 586\,\mathrm{MeV} + \frac{2}{3}\delta\Theta_{1,\bar{c}} + \mathcal{O}\left(\frac{1}{N_c^2}\right),
\end{aligned}
\tag{5.8}
$$

for the isotriplet and isotensor charmed exotics. The $(J_Q \cdot J_q)/N_c m_Q$ hyperfine splitting $\Sigma_c^* - \Sigma_c = 65.6\,\mathrm{MeV}$ also violates heavy quark spin-symmetry, and gives an estimate of the size of $\delta\Theta_{1,\bar{c}}$. The $1/N_c$ expansion does not allow one to predict the mass of the $\Theta_{\bar{c}}$ to great accuracy; a relation such as $\Theta_{\bar{c}} - \Lambda_c = \Theta - \Lambda$ has an order $1/N_c$ correction, and gives a $\Theta_{\bar{c}}$ mass of 2700 MeV with corrections of order $1/N_c$. An order $1/N_c$ mass term is typically of order 100 MeV. The recently reported charm pentaquark at 3099 MeV [32] is 400 MeV above the estimated $\Theta_{\bar{c}}$ mass, and so is unlikely to be the ground state pentaquark. Other estimates of the ground state charm pentaquark mass [20, 33] also give a value much smaller than 3099 MeV. If we identify the resonance at 3099 MeV with one of the excited $\Theta_{1,\bar{c}}$ pentaquarks, then the mass of the ground state charm pentaquark $\Theta_{\bar{c}}$ is more accurately given by $2904^{+(2/3)\delta\Theta_{1,\bar{c}}}_{-(1/3)\delta\Theta_{1,\bar{c}}}$ MeV, depending on whether the observed $\Theta_{1,\bar{c}}$ pentaquark has $j = 1/2$ or $3/2$. This mass result has a $1/N_c^2$ correction, which is estimated to be 30 MeV.

5.2 Widths

The decay widths of the heavy pentaquarks can be studied using the $1/N_c$ expansion and heavy quark symmetry. Heavy pentaquarks can decay within the heavy pentaquark tower by pion emission and to normal baryons plus heavy pseudoscalar and vector mesons $P_{\bar{Q}}$ and $P^*_{\bar{Q}}$.

The transition amplitudes for pion decays of heavy pentaquarks are equal to those of the corresponding light pentaquarks, e.g. $p^{-3}\Gamma(\Theta_{1,\bar{Q},j=1/2} \to \Theta_{\bar{Q}}\pi) = p^{-3}\Gamma(\Theta_{1,j=1/2} \to \Theta\pi)$. One can also use eq. (4.1) to write these as relations between the heavy pentaquark and Δ widths, e.g. $p^{-3}\Gamma(\Theta_{1,\bar{Q},j=1/2} \to \Theta_{\bar{Q}}\pi) = (4/3)p^{-3}\Gamma(\Delta \to N\pi)$. One expects the former version to have smaller $1/N_c$ corrections.

The decay widths $\Gamma(\Theta_{1,\bar{c},j=1/2} \to \Theta_{\bar{c}}\pi)$ and $\Gamma(\Theta_{1,\bar{c},j=3/2} \to \Theta_{\bar{c}}\pi)$ are equal to $0.26\,\Gamma(\Delta \to N\pi) \sim 30\,\text{MeV}$, using 195 MeV for the mass difference between the $\Theta_{1\bar{c}}$ and $\Theta_{\bar{c}}$ states. The two decay rates are equal by heavy quark symmetry, since the decays involve transitions among the light degrees of freedom, and do not affect the heavy quark. The hyperfine splitting $\delta\Theta_{1,\bar{c}}$ violates heavy quark spin symmetry and changes the widths, since it changes the p^3 phase-space dependence. The hyperfine interaction is much smaller for the heavier pentaquarks, and so is less important than in the light pentaquark sector. The $\Theta_{\bar{c},2} \to \Theta_{\bar{c},1}\pi$ widths are extremely broad, and the states will not be observable.

The decay widths of heavy pentaquarks $\Theta_{\bar{Q}}$ to nucleon plus pseudoscalar and vector mesons, $N + P_{\bar{Q}}$ and $N + P^*_{\bar{Q}}$, can be related using heavy quark spin-symmetry and the $1/N_c$ expansion. We first determine the relations which follow from heavy quark symmetry *alone*, and then combine them with the $1/N_c$ results.

Heavy quark spin symmetry relations are derived between the various decay amplitudes. Whether these are allowed decays depends on the masses of the states. The charm pentaquark reported by H1 can decay into both $N + \bar{D}$ and $N + \bar{D}^*$. If it is an excited pentaquark, then some of these decay channels may be kinematically forbidden for the $\Theta_{\bar{c}}$, which is lighter. It is also possible that the $\Theta_{\bar{c}}$ has no strong decays, and only decays via the weak interactions [20, 33]. The $\Theta_{2,\bar{Q}}$ states are likely to be too broad to be seen, so we derive results only for the $\Theta_{\bar{Q}}$ and $\Theta_{1,\bar{Q}}$.

The $\Theta_{\bar{Q}}$ has $J = 1/2$ and $J_\ell = 0$, where J_ℓ is the spin of the light degrees of freedom. It is represented by the spinor field u_Θ. The $\Theta_{1,\bar{Q}}$ states have $J = 1/2, 3/2$ and $J_\ell = 1$, and are represented by the spinor field [34]

$$u^\mu_{\Theta,1} = \frac{1}{\sqrt{3}}\left(\gamma^\mu + v^\mu\right)\gamma_5 u_{\Theta,1,j=1/2} + u^\mu_{\Theta,1,j=3/2}\,. \tag{5.9}$$

The pseudoscalar and vector mesons are represented by the $H^{(\bar{Q})}$ field [35]. To simplify the notation, in any given decay, the initial pentaquark is denoted by Θ, and the final meson by H.

The decay amplitudes are computed in the rest frame of the decaying pentaquark, so that $v = (1, 0, 0, 0)$, and the momentum of the initial state is $p_\Theta = m_\Theta v$. The decay products are non-relativistic, so the velocity of the heavy quark in the heavy meson is also chosen to be v. The momentum of the heavy meson is given by $p_H = m_H v + k$, where k is the residual momentum. The momentum of the nucleon is p_N. Momentum conservation implies that $m_\Theta v = m_H v + k + p_N$, so we have two independent four-vectors, which can be chosen to be v and p_N.

The most general invariant amplitude for $\Theta_{\bar{Q}} \to N + H_{\bar{Q}}$ consistent with heavy quark symmetry is $\bar{u}_N\,\Gamma\,\bar{H}^{(\bar{Q})}\,u_\Theta$ where Γ is an arbitrary bispinor matrix which can be constructed out of v, p_N and products of γ matrices, and u_N is the nucleon spinor. Using $\not{p}_N u_N =$

$m_N u_N$ and $\not{v} H^{(\bar{Q})} = H^{(\bar{Q})}$, one can reduce all the terms to a single invariant with the correct parity and time-reversal properties,

$$f_0 \; \bar{u}_N \, \bar{H}^{(\bar{Q})} \, u_\Theta \tag{5.10}$$

where f_0 is an unknown constant. That there is only one invariant follows by counting the ways in which the spins of the light degrees of freedom in the final state can be combined to form $J_\ell = 0$.

Similarly, for the $\Theta_{1,\bar{Q}}$ states, one can show that there are only two invariants, which are chosen to be

$$\left[f_1 \, \bar{u}_N \gamma_\mu \gamma_5 \bar{H}^{(\bar{Q})} + f_2 \, \bar{u}_N p_{N,\mu} \gamma_5 \bar{H}^{(\bar{Q})}\right] u^\mu_\Theta \tag{5.11}$$

where u^μ_Θ is given in eq. (5.9). A third possible invariant involving the ϵ symbol exists, but is not linearly independent,

$$(m_N + p_N \cdot v)\bar{u}_N \gamma_\mu \gamma_5 \bar{H}^{(\bar{Q})} u^\mu_\Theta = i r \epsilon_{\alpha\beta\nu\mu} v^\alpha p_N^\beta \bar{u}_N \gamma^\nu H^{(\bar{Q})} u^\mu_\Theta + \bar{u}_N p_{N,\mu} \gamma_5 \bar{H}^{(\bar{Q})} u^\mu_\Theta \,. \tag{5.12}$$

The six possible decays of the $\Theta_{\bar{Q}}$ and $\Theta_{1,\bar{Q}}$ into the $P_{\bar{Q}}$ and $P^*_{\bar{Q}}$ mesons are given in terms of three coupling constants $f_{0,1,2}$,

$$\begin{aligned}
\Gamma\left(\Theta_{\bar{Q}} \to P_{\bar{Q}} + N\right) &= \frac{|\mathbf{p}_N| \left(E_N - m_N\right) E_D}{2\pi m_\Theta} f_0^2 \,, \\
\Gamma\left(\Theta_{\bar{Q}} \to P^*_{\bar{Q}} + N\right) &= \frac{|\mathbf{p}_N| \left(E_N - m_N\right) E_D}{2\pi m_\Theta} 3 f_0^2 \,, \\
\Gamma\left(\Theta_{1,\bar{Q},j=1/2} \to P_{\bar{Q}} + N\right) &= \frac{|\mathbf{p}_N| \left(E_N - m_N\right) E_D}{2\pi m_\Theta} \frac{1}{3} \left[3 f_1 + \tilde{f}_2\right]^2 \,, \\
\Gamma\left(\Theta_{1,\bar{Q},j=3/2} \to P_{\bar{Q}} + N\right) &= \frac{|\mathbf{p}_N| \left(E_N - m_N\right) E_D}{2\pi m_\Theta} \frac{1}{3} \tilde{f}_2^2 \,, \\
\Gamma\left(\Theta_{1,\bar{Q},j=1/2} \to P^*_{\bar{Q}} + N\right) &= \frac{|\mathbf{p}_N| \left(E_N - m_N\right) E_D}{2\pi m_\Theta} \left[f_1^2 + \frac{2}{3} f_1 \tilde{f}_2 + \tilde{f}_2^2\right] , \\
\Gamma\left(\Theta_{1,\bar{Q},j=3/2} \to P^*_{\bar{Q}} + N\right) &= \frac{|\mathbf{p}_N| \left(E_N - m_N\right) E_D}{2\pi m_\Theta} \left[4 f_1^2 + \frac{8}{3} f_1 \tilde{f}_2 + \tilde{f}_2^2\right] ,
\end{aligned} \tag{5.13}$$

where $\tilde{f}_2 = (m_N + E_N) f_2$, and we have normalized the amplitudes in eqs. (5.9), (5.10) so that the isospin Clebsch-Gordan factors are unity for each decay. There are three relations among the decay rates, which follow from heavy quark spin symmetry (and do not use the $1/N_c$ expansion),

$$\begin{aligned}
p^{-3}\Gamma\left(\Theta_{\bar{Q}} \to P^*_{\bar{Q}} + N\right) &= 3 p^{-3}\Gamma\left(\Theta_{\bar{Q}} \to P_{\bar{Q}} + N\right) , \\
p^{-3}\Gamma\left(\Theta_{1,\bar{Q},j=1/2} \to P^*_{\bar{Q}} + N\right) &= \frac{1}{3} p^{-3}\Gamma\left(\Theta_{1,\bar{Q},j=1/2} \to P_{\bar{Q}} + N\right) + \\
&\quad + \frac{8}{3} p^{-3}\Gamma\left(\Theta_{1,\bar{Q},j=3/2} \to P_{\bar{Q}} + N\right) , \\
p^{-3}\Gamma\left(\Theta_{1,\bar{Q},j=3/2} \to P^*_{\bar{Q}} + N\right) &= \frac{4}{3} p^{-3}\Gamma\left(\Theta_{1,\bar{Q},j=1/2} \to P_{\bar{Q}} + N\right) + \\
&\quad + \frac{5}{3} p^{-3}\Gamma\left(\Theta_{1,\bar{Q},j=3/2} \to P_{\bar{Q}} + N\right) ,
\end{aligned} \tag{5.14}$$

where we have used $E_N - m_N \approx p_N^2/(2m_N)$, since the nucleon is non-relativistic. In these relations p is the three-momentum of the final state particle for the corresponding decay. These relations are violated by $1/m_Q$ corrections.

In addition to the heavy quark relations eq. (5.14), there are relations which follow from isospin symmetry. The Θ_Q is isospin zero, and so

$$\Gamma\left(\Theta_{\bar{Q}} \to H(d\bar{Q}) + p\right) = \Gamma\left(\Theta_{\bar{Q}} \to H(u\bar{Q}) + n\right) = \frac{1}{2}\Gamma\left(\Theta_{\bar{Q}} \to H_{\bar{Q}} + N\right) \tag{5.15}$$

where $H(q\bar{Q})$ represents either the pseudoscalar or the vector meson with quark content $q\bar{Q}$, and the right hand side is the total width into the corresponding channel. The $\Theta_{1,\bar{Q}}$ has isospin one, so the $I_3 = 1$ pentaquark decays entirely into $H(u\bar{Q}) + p$, the $I_3 = -1$ pentaquark decays into $H(d\bar{Q}) + n$ and the $I_3 = 0$ pentaquark decays 50% of the time into $H(d\bar{Q}) + p$ and 50% of the time into $H(u\bar{Q}) + n$.

The $1/N_c$ relations for the heavy pentaquark decays into pseudoscalar mesons are identical to the relations in eq. (4.1), with the replacement $K \to P_{\bar{Q}}$. Combining the heavy quark predictions eq. (5.14) with the large N_c relations gives the decay width ratios:

$$\begin{aligned}
p^{-3}\Gamma\left(\Theta_{\bar{Q}} \to P_{\bar{Q}} + N\right) : p^{-3}\Gamma\left(\Theta_{\bar{Q}} \to P^*_{\bar{Q}} + N\right) : p^{-3}\Gamma\left(\Theta_{1,\bar{Q},j=1/2} \to P_{\bar{Q}} + N\right)& \\
: p^{-3}\Gamma\left(\Theta_{1,\bar{Q},j=3/2} \to P_{\bar{Q}} + N\right) : p^{-3}\Gamma\left(\Theta_{1,\bar{Q},j=1/2} \to P^*_{\bar{Q}} + N\right)& \\
: p^{-3}\Gamma\left(\Theta_{1,\bar{Q},j=3/2} \to P^*_{\bar{Q}} + N\right) =& \\
= 1 : 3 : \frac{2}{9} : \frac{8}{9} : \frac{22}{9} : \frac{16}{9}\,.&
\end{aligned} \tag{5.16}$$

These relations are violated by $1/N_c^2$ and $1/m_Q$ corrections.

In the large-N_c limit, the method of ref. [11] gives the heavy pentaquark-baryon-heavy meson couplings in terms of a single coupling. The large-N_c spin-flavor symmetry implies that $\tilde{f}_2 = -2f_1$ and $f_1 = \sqrt{2/3}f_0$, which gives another way to derive eq. (5.16).

One can similarly work out the consequences of heavy quark symmetry for the decays of negative parity heavy pentaquark states. The decay amplitudes are given by replacing eq. (5.10) by $f_0\, \bar{u}_N\, i\gamma_5\, \bar{H}^{(\bar{Q})}\, u_\Theta$ and dropping the γ_5 in both terms in eq. (5.11). The decay widths are given by the replacement $m_N \to -m_N$ in eq. (5.13) and in the definition of $\tilde{f}_2$. The phase space is now proportional to $|\mathbf{p}|$, since the decays are predominantly s-wave. The relations eq. (5.14) are replaced by

$$\begin{aligned}
p^{-1}\Gamma\left(\tilde{\Theta}_{\bar{Q}} \to P^*_{\bar{Q}} + N\right) &= 3p^{-1}\Gamma\left(\tilde{\Theta}_{\bar{Q}} \to P_{\bar{Q}} + N\right), \\
p^{-1}\Gamma\left(\tilde{\Theta}_{1,\bar{Q},j=1/2} \to P^*_{\bar{Q}} + N\right) &= \frac{1}{3}p^{-1}\Gamma\left(\tilde{\Theta}_{1,\bar{Q},j=1/2} \to P_{\bar{Q}} + N\right), \\
p^{-1}\Gamma\left(\tilde{\Theta}_{1,\bar{Q},j=3/2} \to P^*_{\bar{Q}} + N\right) &= \frac{4}{3}p^{-1}\Gamma\left(\tilde{\Theta}_{1,\bar{Q},j=1/2} \to P_{\bar{Q}} + N\right), \\
p^{-1}\Gamma\left(\tilde{\Theta}_{1,\bar{Q},j=3/2} \to P_{\bar{Q}} + N\right) &= \mathcal{O}\left(|\mathbf{p}_N|^4\right),
\end{aligned} \tag{5.17}$$

since $\tilde{f}_2 \approx |\mathbf{p}_N|^2 f_2/(2M_N)$ is small. We have denoted the negative parity states by $\tilde{\Theta}_{\bar{Q}}$ and $\tilde{\Theta}_{1,\bar{Q}}$, which have $J_\ell = 0$ and $J_\ell = 1$, respectively. The last decay rate is $\mathcal{O}(|\mathbf{p}_N|^4)$, since it is a d-wave decay.

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